The Voronoi Diagram of Rotating Rays

with applications to Floodlight Illumination

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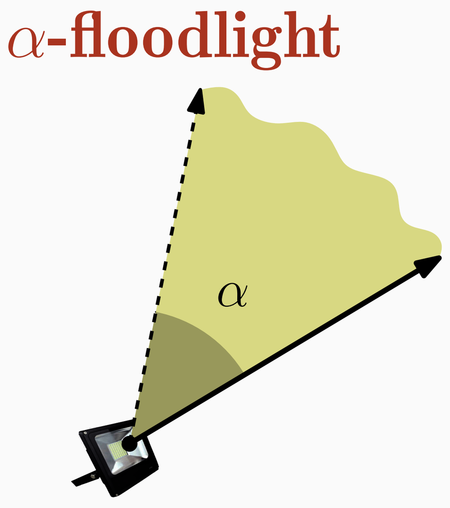
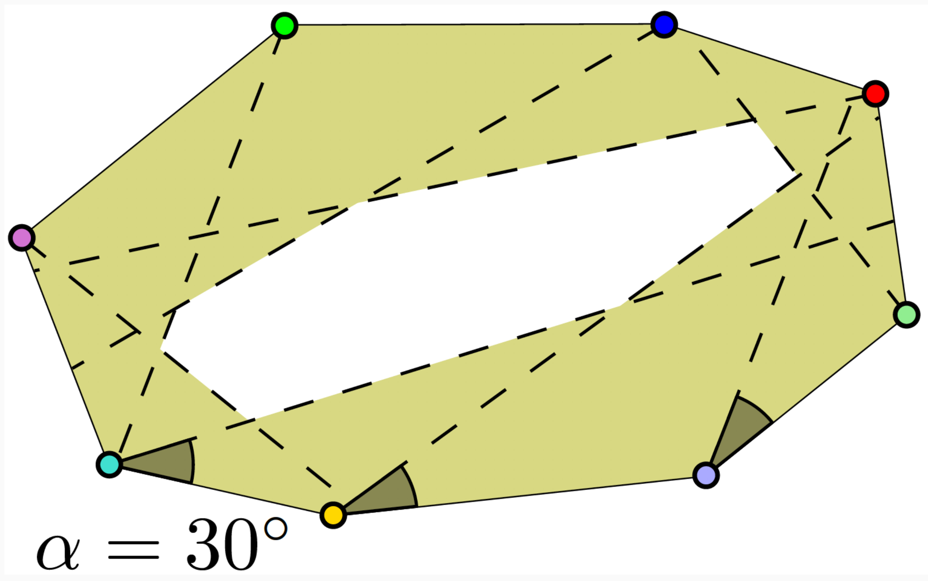
Brocard Illumination Problem



Problem

Given a polygon P with an α-floodlight aligned at each edge, what is the **minimum angle** α^* needed to illuminate P?
α^* is the **Brocard angle** of P.

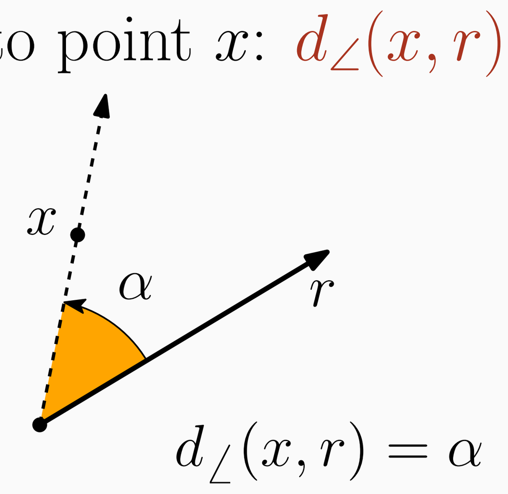


Approach

We define the **Rotating Rays Voronoi Diagram (RVD)**. The sites are **rays** and the distance is the **angular distance**. The Brocard angle is realized at a vertex of the RVD.

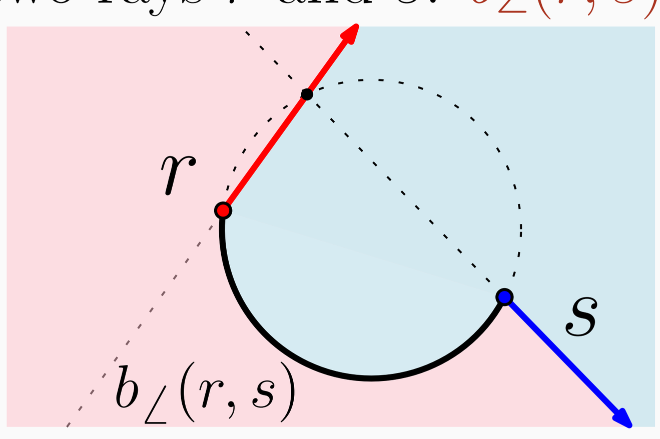
Angular Distance

The **angular distance** of a ray r to point x: $d_\angle(x, r)$.

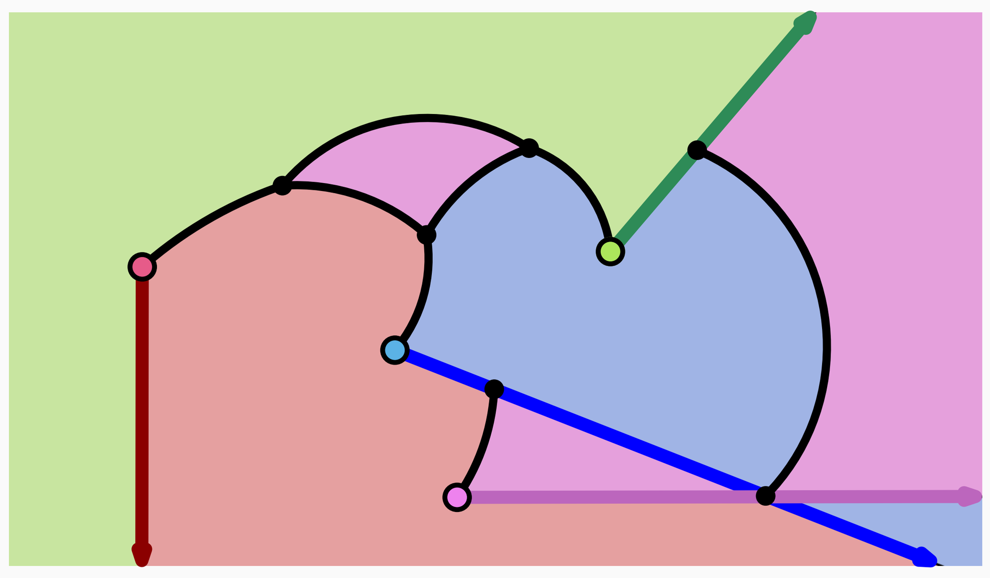


Angular Bisector

The **angular bisector** of two rays r and s: $b_\angle(r, s)$.

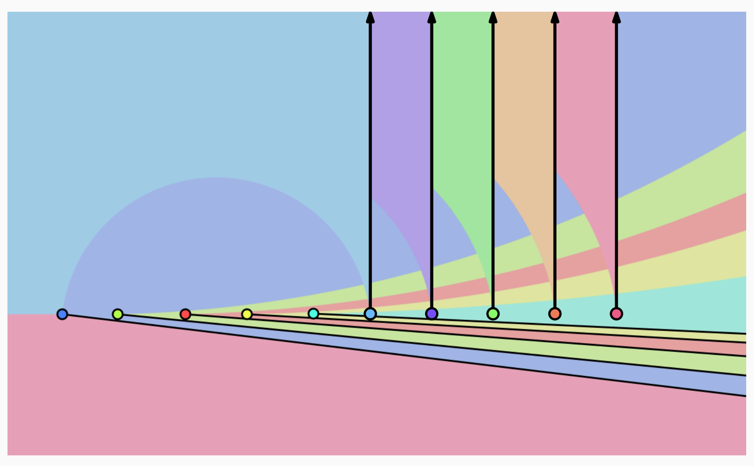


Rotating Rays Voronoi Diagram



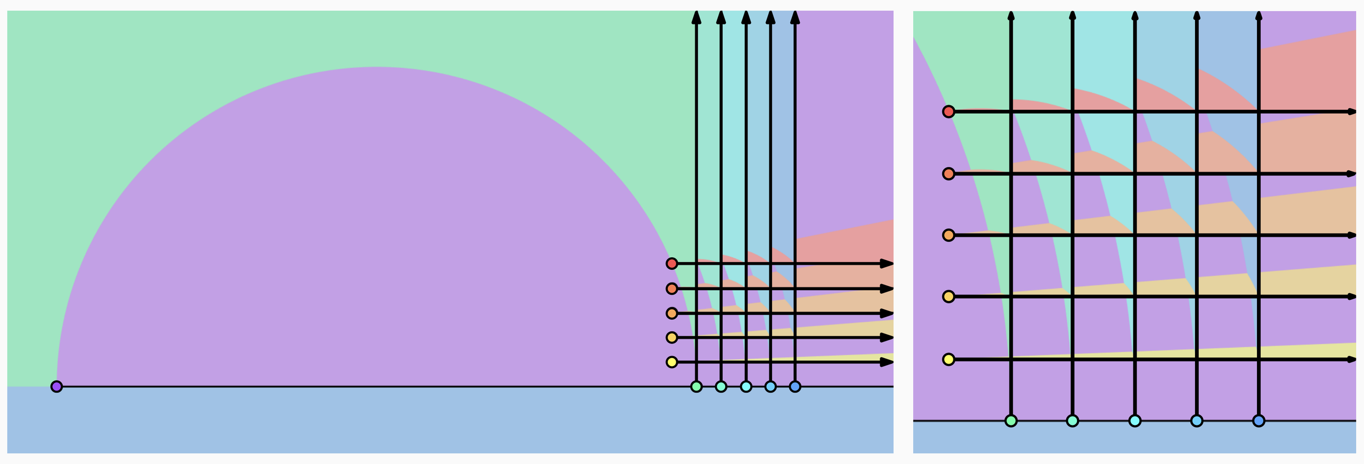
The **Voronoi region** of $r \in \mathcal{S}$ is the locus of points closer to r. The **Rotating Rays Voronoi Diagram** of $\mathcal{S}$ is the subdivision of $\mathbb{R}^2$ in Voronoi regions.

Rotating Rays Voronoi Diagram in $\mathbb{R}^2$



Theorem

RVD($\mathcal{S}$) has $\Omega(n^2)$ **worst case complexity** even if the rays are non-intersecting.



Theorem

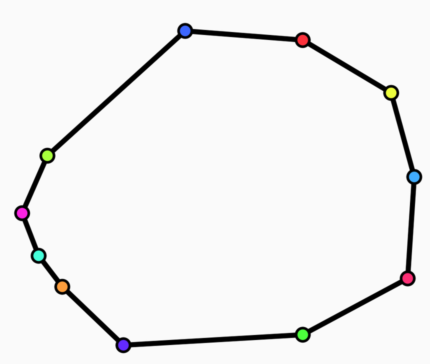
A single Voronoi region of RVD($\mathcal{S}$) can have $\Theta(n^2)$ **complexity** in the worst case.

Theorem

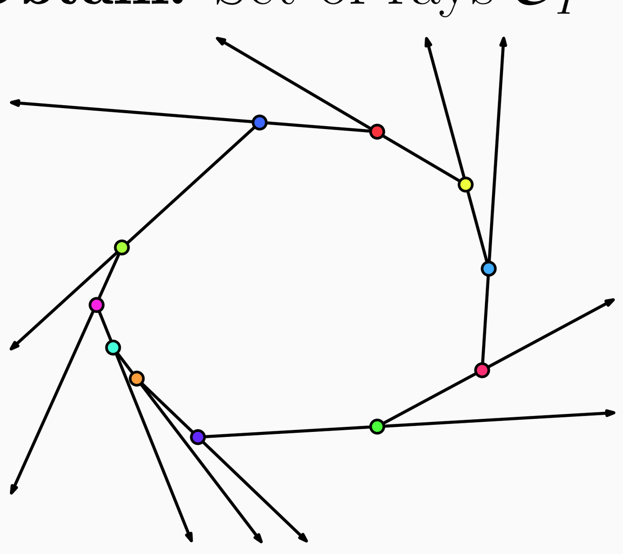
RVD($\mathcal{S}$) has $O(n^{2+\epsilon})$ **complexity** and it can be constructed in $O(n^{2+\epsilon})$ **time**.

Rotating Rays Voronoi Diagram of a Convex Polygon

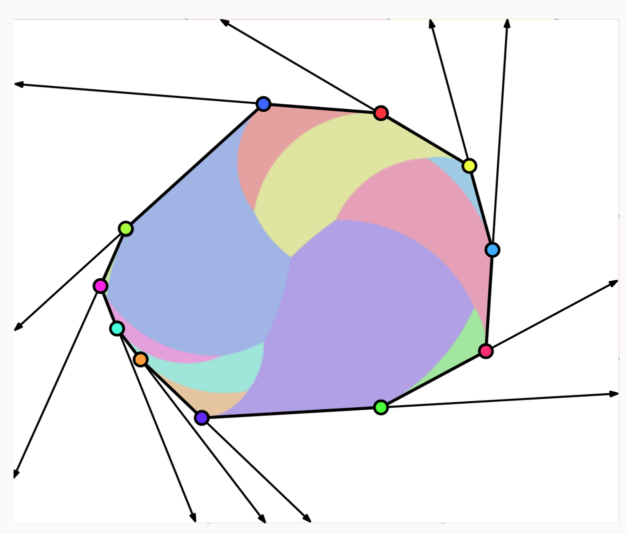
Input: Convex polygon P



Obtain: Set of rays $\mathcal{S}_P$



Output: Diagram PRVD($\mathcal{S}_P$)



Algorithm outline

1. Parition $\mathcal{S}_P$ in 4 sets.
2. Construct each of the 4 diagrams.
3. Merge the 4 diagrams.

Theorem

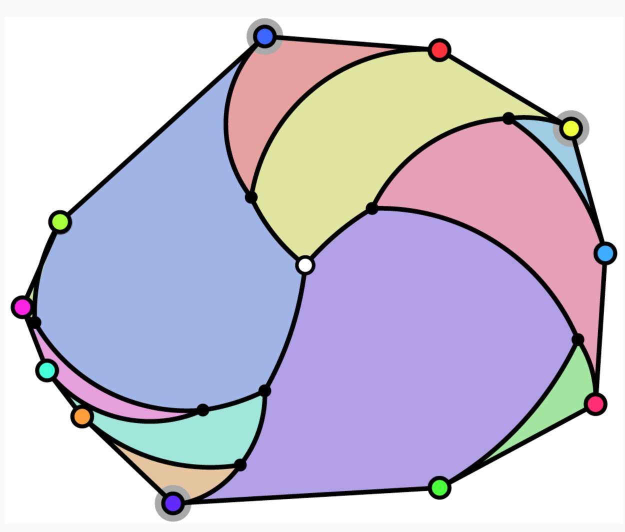
PRVD($\mathcal{S}_P$) can be constructed in $\Theta(n)$ **time**.

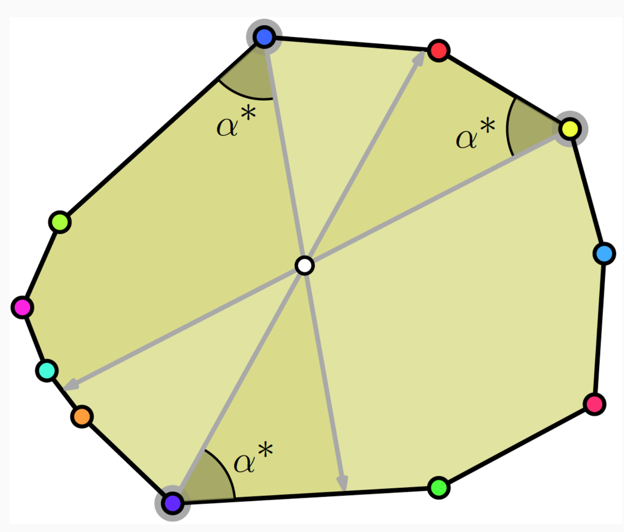
Finding the Brocard angle

Brocard angle of polygon P

1. Create PRVD($\mathcal{S}_P$)
2. Traverse PRVD($\mathcal{S}_P$)

- **Three α^*-floodlights suffice**
- $\alpha^* \in (0, \pi/2 - \pi/n]$





Brocard angle of $\mathcal{S}$ in $\mathbb{R}^2$

1. Create RVD($\mathcal{S}$)
2. Traverse RVD($\mathcal{S}$)

- α^* realized at vertex of RVD($\mathcal{S}$)
- $\alpha^* \in (2\pi/n, 2\pi)$